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Effect of the low-order coefficients of the Earth gravity model in calculating the satellite orbit



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ABSTRACT

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The Earth's gravity model is a crucial factor in determining satellite orbits. Scientific organizations such as GFZ Potsdam in Germany, GRGS Toulouse in France, and AIUB in Switzerland have established Earth gravity models with increasing precision of spherical harmonic coefficients. Low-order coefficients, including \overline{C}_{21} , \overline{S}_{21} , \overline{C}_{10} , \overline{C}_{11} , \overline{S}_{11} and \overline{C}_{20} , play a vital role in describing changes in the Earth's poles, geometric center, and flattening. To evaluate the impact of these coefficients and understand altimeter satellite orbital error, the Propagerror program was developed. This program calculates satellite orbital error from the differential components of Earth gravity model spherical harmonic coefficients (dC/Slm), which can be obtained from the difference between two gravity models or from seasonal and annual components of spherical harmonics. By separating appropriate low-order components in the Earth gravity models, the Propagerror program enables the estimation of satellite orbital error. In this study, we isolate \overline{C}_{10} , \overline{C}_{11} , \overline{S}_{11} , and \overline{C}_{21} , \overline{S}_{21} coefficients in the EIGEN-GRGS.RL02bis.MF and EIGEN-6S gravity models to assess the geophysical impact on satellite orbits. The influence of the geometry center elements results in a 2 cm error in the Jason-2 satellite, while the rotational axis elements have no effect. The \overline{C}_{31} , \overline{S}_{31} coefficient has a 6-7 mm impact on the accuracy of the Jason-2 satellite, as demonstrated_by the satellite error map in two situations with and without the \bar{C}_{31} , \bar{S}_{31} harmonic coefficient. This study highlights the significance of regulating function coefficients in satellite orbit determination, particularly the low-level harmonic parameters. The Propagerror program provides insights into the impact of each spherical harmonic parameter on satellite orbits, contributing to the improvement of orbit accuracy and the understanding of the Earth's gravity model.

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1. Introduction

**Corresponding author E - mail:* trongtd@huce.edu.vn DOI: 10.46326/JMES.2023.64(3).10 The Earth's gravity model plays a crucial in studying space satellite orbit (Rummel, 2020). As described by (Barthelmes, 2013), the Earth's

gravity field can be modeled mathematically through spherical harmonic coefficients and is expressed by equation (1).

$$V(r,\varphi,\lambda) = \frac{GM}{r} \sum_{l=0}^{l_{max}} \sum_{m=0}^{l} \left(\frac{R}{r}\right)^{l} \bar{P}_{lm}(sin\varphi)(\bar{C}_{lm}cosm\lambda) + \bar{S}_{lm}sinm\lambda$$
(1)

In which: V is the gravitational potential; r, φ , and λ correspond to the spherical geocentric coordinates of the computation point (radius, latitude, and longitude); R is a reference radius, usually taken as the mean semi-major axis of Earth in geodesy; GM is the gravitational constant times the mass of the Earth; l and m are degree and order of spherical harmonic, l_{max} is the maximum degree of the model expansion; \bar{P}_{lm} are fully normalized Legendre polynomials of degree l and order m, and \bar{C}_{lm} and \bar{S}_{lm} are spherical harmonic coefficients.

The low-degree and low-order spherical harmonic functions have well-defined physical interpretations. The spherical harmonic coefficient \bar{C}_{00} describes the mass of the Earth, scaling the value of GM, which is the product of the gravitational constant and the Earth's mass. Its value is close to 1. The degree 1 spherical harmonic coefficients \bar{C}_{10} , \bar{C}_{11} , and \bar{S}_{11} are related to the coordinates of the Earth's geometric center, and they are equal to zero if the origin of the coordinate system coincides with the geometric center. The coefficients \bar{C}_{21} and \bar{S}_{21} are related to the mean position of the rotational pole (Ince et al., 2019). Understanding the low-order coefficients of the Earth's gravity model enables us to consider the key factors that impact satellite orbital error. The Earth's gravity model is composed of polynomials and the regulating functions cosine/sine (C/S). The coefficients \bar{C}_{10} and \bar{C}_{11} , \bar{S}_{11} are geometrically defined, \bar{C}_{20} represents flattening, and \bar{C}_{21} , \bar{S}_{21} describe changes in the Earth's geometry axis (Luong, 2015)

Using laser measurement values from satellites such as LAGEOS (Laser Geodynamics Satellite) 1 and 2 (Cohen & Smith, 1985) and Starlette (Zelensky et al., 2014), Gourine (2012) analyzed the coordinate time series of ground points and Earth parameters

(EOP - Earth Orientation Parameters) to demonstrate the Earth's geometrical coefficients fluctuate by 4-5mm per year, which is relate to changes in seasonal and annual surface material distribution. In the study on the determination of the Jason-2 satellite orbit by GPS (Global Positioning System), Bertiger et al. (2010) found that the Z-axis component error of the Earth's center produces an error of 4+5 mm on the satellite orbit, taking into account the influence of the Earth's center. Sośnica et al. (2012) demonstrated that considering only linear components and constant components of \bar{C}_{20} coefficients in the process of accurately defining LAGEOS satellite orbit is insufficient for secondorder spherical harmonic coefficients, as all the components of spherical harmonic coefficients (constant, linear and instantaneous) must be considered to fully calculate the effect of Earth flattening on satellite trajectory error. To accurately calculate the effect of the Earth flattening factor on satellite trajectory error, all constant components, linear components (longterm variation), and instantaneous components (seasonal or annual variation) of the spherical harmonic coefficient must be considered. Additionally, several authors have explored the of low-level spherical impact harmonic coefficients on satellite orbits, such as Zelensky et al. (2014), who calculated the effect of coefficients $\overline{C}_{22}, \overline{S}_{22}$ and $\overline{C}_{31}, \overline{S}_{31}$ on the high orbit of the SARAL (Satellite with ARgos and ALtiKa) satellite (Verron et al., 2015) and found that the \bar{C}_{31} , \bar{S}_{31} coefficients are highly sensitive to the SARAL satellite orbit. In a separate study, Couhert et al. (2015) argued that adjusting the harmonic coefficients \bar{C}_{31} , \bar{S}_{31} would improve the stability of the calculations for the Jason-2 satellite orbit.

As evidence, regulation of function coefficients in satellite orbit determination, particularly low-level harmonic parameters, is of utmost importance. To address this issue, the Propagerror program was developed, which is described in Section 2. This program is based on the satellite motion formula according to either the 6 Keplerian parameters (Seeber, 2003) and the algorithm established by Bois (1994) established by Exertier and Bonnefond (1997). The program enables the evaluation of the various sources of influence on satellite orbits through global image maps of satellite orbits. Additionally, the program allows for the separation of spherical harmonic parameters from the Earth gravity model while computing orbital accuracy, providing insight into the impact of each spherical harmonic parameter on the satellite orbit.

The introduction to this paper is provided in Section 1, while the computational program is described in detail in Section 2. The study results and evaluations are presented and discussed in Sections 3 and 4, respectively.

2. Propagerror Program

Adhering to three Kepler's laws (Kaula, 1966; Seeber, 2003), the orbits of artificial satellites can be described using the 6 Keplerian parameters (including the semi-major axis of the satellite orbit, eccentricity, orbit inclination with respect to the equatorial plane, the latitude of the ascending node in the equatorial plane, argument of periapsis, and mean anomaly). When the eccentricity element is close to zero, corresponding to a quasi-circular orbit, the calculation of the orbit error becomes tractable. In light of this, Exertier and Bonnefond (1997) developed a formula for calculating the quasicircular orbital error based on the geographic coordinate system parameters (r, φ , λ) and the geometric equations and motion of Bois (1994).

Building on these theories, we developed the Propagerror program, which includes the following basic calculation steps:

1. The program calculates the average orbital elements based on the position and velocity of the satellite in the ephemeris. The program enables the selection of orbital calculations using either



Figure 1. The average Saral satellite trajectory (brown-dashed) and Jason-2 (blue-solid).

the 6 satellite orbital parameters (a, e, i, ω , Ω , M) or the geographical coordinates (r, φ , λ), depending on the nature of the satellite. A filtering process of satellite calendars is used to determine the epoch for the average orbital elements. The satellite ephemeris is also involved in the calculation process, but it only assists in determining the projection of the average orbit on a global map (Figure 1).

2. According to the theory of spatial geodesy error, the first-class differential components of spherical harmonic parameters are considered Earth gravity errors. The Propagerror program filters errors with an amplitude greater than a specified limit threshold. Then, the averaged satellite orbit coordinates are used to interpolate the corresponding errors in the satellite's local coordinate system (according to the radial axis -R, centrifugal - T, and perpendicular to the orbit plane - N).

3. The output results are used to extract the amplitude and frequency of these errors, and the image of the error distribution around the averaged satellite orbit can be represented (Bayen & Siauw, 2015).

The flowchart of the Propagerror program is presented in Figure 2. The program is capable of computing the error in satellite orbits using the differential components of the spherical harmonic coefficients of the Earth's gravity model (dC/Slm). These derivative components can be obtained from the difference between two gravitational models or by extracting seasonal and annual components the spherical of harmonic coefficients. To enhance the efficiency of the program, two algorithms have been incorporated, namely, Kaula's algorithm (Kaula, 1966) and Exertier's algorithm (Exertier & Bonnefond, 1997). These algorithms differ fundamentally in the input parameters used for satellite orbit calculation.

For Kaula's algorithm, the input parameters are 6 Keplerian parameters. The eccentricity must be equal to or greater than 0.003 for elliptical satellites and less than 0.003 for circular or near-circular satellite orbits. On the other hand, for the Exertier's algorithm, the input parameters are the spatial coordinates of the satellite (r, φ , λ). The program allows for the calculation of orbital



Figure 2. Flowchart of Propagerror program.

errors in a more efficient manner by incorporating these two algorithms.

The Propagerror program includes features that facilitate the separation of low-level harmonic coefficients in the calculation process. For instance, the option "G" (geocenter) can be utilized to isolate the coefficients \bar{C}_{10} and $\bar{C}_{11}, \bar{S}_{11}$, while option "P" is for isolating the coefficients $\bar{C}_{21}, \bar{S}_{21}$. Additionally, the program has the ability to isolate any harmonic coefficients from the first order to the tenth level and consider their individual effect.

3. Results

In this section, the results of the analysis of the impact of Earth gravity models on the satellites Jason-2 (Lambin et al., 2010) and SARAL are presented. The study utilized two Earth gravity models, namely EIGEN-GRGS.RL02bis.MF and EIGEN-6S (F. Flechtner et al., 2010), which considered spherical harmonic coefficients from orders 1 to 50 and their differential.

Sośnica et al. (2012) conducted a study on the influence of eleven gravity models and found that the orbit of the LAGEOS satellite at an altitude of 6000 km is only impacted by harmonic coefficients from steps 1 to 14 of the Earth gravity model. This result demonstrates the dependence of the impact of the gravity model on the height of the satellite orbit. It should be noted that the Jason-2 satellite, flying at an altitude of 1335 km, was influenced by 1935 parameters, while the SARAL satellite, at an altitude of 780 km, was



Figure 3. Filtering of harmonic coefficients (black) and, the differential (blue) of the EIGEN-GRGS.RL02bis.MF model.

influenced by 3,660 parameters. Although Figure 3 does not depict these numbers explicitly, it is apparent that the parameter density of the SARAL satellite is higher than that of the Jason-2 satellite.

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Figure 3 demonstrates the filtering of harmonic coefficients (represented in black) and the differential (represented in blue) of the EIGEN-GRGS.RL02bis.MF model, which has an effect greater than 1 cm on the satellite orbits of Jason-2 and SARAL. The horizontal axis in the figure represents the period of the harmonic coefficients, while the vertical axis represents their spectral values. Furthermore, the results displayed in Figure 3 highlight the ability to evaluate the effect of individual coefficients (ranging from levels 1 to 10) on the satellite orbit. This is achieved by isolating the \bar{C}_{31} , \bar{S}_{31} coefficients and differential components for analysis of their impact on the orbit of the Jason-2 satellite.

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In order to comprehend the influence of geophysical factors (the axis of rotation and geometry center of Earth) on the satellite orbit error, we first identify their geographical distribution on the global geoid map (Figure 4). The top image in Figure 4 depicts a geoid map without the inclusion of geometrical center elements (\bar{C}_{10} , \bar{C}_{11} , \bar{S}_{11}) and the axis of rotation (\bar{C}_{21} , \bar{S}_{21}). The second map includes all these elements, while the third map only includes the geometrical center elements, and the last map only includes the rotating axis component.



Figure 4. Global geoid map to identify the distribution of geometry center factors (correlation coefficient C10, C/S11) and axis of rotation (correlation coefficient of polarity C/S21).

From the global geoid maps shown in Figure 4, it is evident that the effects of the geometrical center and rotating axis components are symmetrical on both North-South hemispheres, with a value of approximately ± 2 cm for the geometrical center components. On the other hand, the rotating axis components are minimal and have a value of less than ± 0.1 mm. This suggests that the geometrical center components play a more significant role than the rotating axis components in calculating the geoid map.

In order to understand the impact of firstand second-order harmonic coefficients on satellite orbit error, we conducted an experiment using the Jason-2 satellite orbit and the EIGEN-GRGS.RL02bis.MF and EIGEN-6S earth gravity models. The results are shown in Figure 5, which displays the radial error component R of the Jason-2 satellite orbit when projected using the EIGEN-GRGS.RL02bis.MF gravity model. The top



Figure 5. The error maps of the Jason-2 orbit (interest component R) under the influence of first-order harmonic coefficients (center component) and second-order (rotational component).

panel of Figure 5 shows the error map when the geoid map does not include \bar{C}_{10} , \bar{C}_{11} , \bar{S}_{11} , and \bar{C}_{21} , \bar{S}_{21} , the second panel includes all their elements, and the third and fourth panels show the error map when the geoid map has only \bar{C}_{10} , \bar{C}_{11} , \bar{S}_{11} , or \bar{C}_{21} , \bar{S}_{21} , respectively. It can be seen that the small effect of rotating axis elements has minimal impact on satellite error, while the influence of geometry center elements can result in an error of ± 2 cm. The third map from top to bottom in Figure 5 reveals that the error is distributed in the southwestern hemisphere, which may be due to the current earth gravity model not being completely accurate.

In this study, we aimed to investigate the effect of first- and second-order harmonic coefficients on the Jason-2 satellite orbit. For this purpose, we utilized the EIGEN-GRGS.RL02bis.MF and EIGEN-6S earth gravity models for calculation. The radial error component R of the Jason-2 satellite orbit was projected onto a plane using the EIGEN-GRGS.RL02bis.MF gravity model, and its distribution maps were depicted in Figure 5.

In Figure 5, the top panel displays the error map of the satellite orbit when the geoid map does not include \bar{C}_{10} , \bar{C}_{11} , \bar{S}_{11} , and \bar{C}_{21} , \bar{S}_{21} . The second panel includes all of these elements, while the third and fourth panels show the error map when the geoid map has only \bar{C}_{10} , \bar{C}_{11} , \bar{S}_{11} , or \bar{C}_{21} , \bar{S}_{21} , respectively.

Our results indicate that the small effect of the rotating axis elements does not significantly affect the satellite error. However, the influence of the geometric center elements can cause an error level of ± 2 cm in the satellite's orbit. The distribution of the satellite error in the southwestern hemisphere as shown in the third map from the top of Figure 5 is different from that displayed on the geoid map. This discrepancy may arise from the fact that the current earth gravity model is not entirely accurate.

Couhert et al. (2015) have demonstrated the impact of harmonic coefficients \bar{C}_{31} , \bar{S}_{31} on satellite orbits, Jason-2 and SA RAL with radial error components R to be 0.42 mm and 0.45 mm, respectively. To assess the effect of \bar{C}_{31} , \bar{S}_{31} on the Jason-2 satellite orbit, we isolated these coefficients from the EIGEN-GRGS.RL02bis.MF model. In another respect, Couhert et al. (2015) have shown that the effects of harmonic coefficients \bar{C}_{31} , \bar{S}_{31} on satellite orbits Jason-2 and SARAL with components of the directional error R are 0.42 and 0.45 mm, respectively. We also isolated coefficients \bar{C}_{31} , \bar{S}_{31} from the EIGEN-GRGS.RL02bis.MF model to calculate the impact on Jason-2 satellite orbit.

Figure 6 shows the radial error component distribution maps (R) of Jason-2 satellite orbit when applying the gravitational model EIGEN-GRGS.RL02bis.MF. The map above shows the error in satellite orbits when the geoid model does not include the harmonic coefficients \overline{C}_{31} , \overline{S}_{31} , while the map below shows the error when the geoid model includes these coefficients. Results indicate that the inclusion of \overline{C}_{31} , \overline{S}_{31} coefficients in the geoid model lead to an error of 6÷7 mm in the Jason-2 satellite orbit. This error



Figure 6. Maps of the radial component error distribution (R) of Jason-2 satellite orbit when using the EIGEN-GRGS.RL02bis.MF gravity model without (above) and with \overline{C}_{31} , \overline{S}_{31} (below). The unit is meters

estimate differs from the study of Couhert et al. (2015), which estimated the effect of \overline{C}_{31} , \overline{S}_{31} on the satellite orbits Jason-2 and SARAL to be 0.42 and 0.45 mm, respectively.

However, we surmise that this difference arises from the method data handling. This hypothesis was verified by stacking satellite error maps to compare the error of the Jason-2 satellite in two scenarios, one with and one without the \bar{C}_{31} , \bar{S}_{31} harmonic coefficient. The resulting difference was approximately 0.5 mm, which is in close agreement with the value of 0.42 mm reported in the study by Couhert et al. (2015).

4. Conclusion

In this study, the importance of evaluating the impact of lower-level harmonic coefficients on satellite orbital errors was investigated. The Propagerror program was utilized to analyze the influence of these coefficients on satellite orbital error by allowing the isolation of each desired harmonic coefficient from level 1 to level 10. Results indicated that the correlation between the effects of low-level harmonic coefficients and the error of the Jason-2 satellite orbits was consistent with previous findings. Although limitations in calculation still exist and the distribution of the effects of harmonic coefficients and the distribution of satellite error were not fully explained, the advantages of separately evaluating the impact of geophysical elements on satellite orbital error were demonstrated.

Future research will focus on the critical lowlevel errors that often impact satellite orbital error, particularly the coefficient \bar{C}_{20} , satellite orbital errors for different Earth gravity models, and the effect of satellite orbital error on the position error of ground station points.

Contribution of authors

Dung Ngoc Luong – writing, designed the study, and checked the results of the satellite orbit analysis. Trong Dinh Tran - analyzed the satellite orbits using Propagerr, and edited the article.

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